Research on the impact of jump volatility on the volatility of Shanghai securities index

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Abstract: The research on stock market volatility has been a hot spot in the financial field, in recent years, with the development of computer technology and the reduction of the cost of information storage, high-frequency data provide a new direction for studying volatility. Based on the high-frequency data of Shanghai securities 50 index, this paper uses the modified realized volatility to replace the real volatility, and uses HAR-RV-CJ model and the extended HAR-RV-CJ model to study the effect of jump on the volatility of Shanghai securities Index. The empirical evidence shows that the extended HAR-RV-CJ model has better ability to explain the volatility; There are many medium-term investors in China’s stock market, and the market is immature and needs to be improved; The existence of jump component has positive effect on the volatility of Shanghai securities 50 Index.

1. Introduction

Financial Market is an indispensable part of a country’s development. Many scholars have done a lot of research on financial market.As the core of asset pricing and risk management, volatility has been the focus of financial research.In the past, many estimation models of volatility have been given, and the results of different models are different, which shows that the existing models still have defects and can not fit the real volatility well.

The early research on volatility mainly focused on GARCH models, including TGARCH, EGARCH, GARCH-MIDAS and so on.The sample data of this kind of model mostly use daily or monthly data. This kind of low frequency data with daily or monthly frequency reduces a lot of market noise, but also loses a lot of daytime information.With the development of computer technology and the reduction of the cost of information storage,high-frequency data provide a new direction for studying volatility.Andersen and Bollerslev[1] were the first to study volatility using high frequency data.They proposed the RV model, in which the sum of the squares of the daytime returns is used as an approximate estimate of volatility.The true volatility can be regarded as the sum of the squares of the return of the infinite sampling frequency.This non-parametric method does not need complicated model calculation and has better fitting effect.Since then, the work of studying volatility with high-frequency data has been greatly expanded.The most representative ones are ARFIMA-RV model of Andersen et al[2] and HAR-RV model of Corsi[3].Both of these models prove the long memory property of the volatility,and have good prediction effect.In contrast to the former, the HAR-RV model can not only capture the long-memory properties of RV, but also explain the fat-tail and right-deviation properties of the return.Moreover, the HAR-RV model is more simple, has the clear economic significance as well as the strong plasticity. Therefore it is more favored by scholars, has become the benchmark model of researching realized volatility. Bandi and Russell [4] show that volatility based on high-frequency returns is not a consistent estimate of true volatility, due to market microstructure noise.Hansen and Lunde[5] propose a method to reduce the noise using kernel estimators so that the volatility can be approximated to the real situation.

As an emerging market, China’s market participants are mostly private investors, and their ability to obtain and capture information is poor.When facing the turmoil of the stock market, it is easy to lead to panic,producing a Herd Effect,chasing up and down, and then intensify the volatility of the stock market,generating jump behavior, and ultimately make the forecast of volatility bias.Therefore, it is important to join the jump variable when studying the volatility of my country's stock market.

Most foreign research on jumping is concentrated on jumping recognition and prediction.There are two main methods, one is parametric method based on low-frequency data, the other is non-parametric method based on high-frequency data.As far as recognition is concerned, the detection method of jumping is very important.Barndorff-Nielsen[6,7] proposed the BNS jump detection method, which uses the difference between realized volatility and realized the quadratic power variation to estimate the jump part,pioneering a pre-river in which no parametric research jump volatility.However, due to the influence of market structure noise in high frequency data, the quadratic power variation is not a consistent estimation of realized volatility, which makes this method lack of robustness.Andersen et al[8] proposed a new method to detect jumps based on the realized volatility of median, which is more robust.As far as prediction is concerned, Andersen et al[9] separated the jump part from the realized volatility by using the quadratic power variation theory, and added it to the model as an explanatory variable to predict the realized volatility. It is found that the explanatory power of volatility mostly comes from the continuous part, while the explanatory power of jump part is weak.Maheu and McCurdy[10] further extended the CARJI model by adding the effects of past jumps, and found that past jumps do not produce asymmetric effects on volatility, whereas current jumps do.Andersen et al[11] used extended Har model to model the continuous part and the jump part.The daily, weekly and monthly effects of continuous volatility are significant, and the daily effects of jump volatility are also significant and the jump interval and jump size have self-correlation.

Due to the late start of the domestic market, there are relatively few related aspects.Zhang Chuanhai[12] used the extended HAR Model to study the ability of noise variance to predict continuous and jump volatility, and found that noise variance can positively predict the volatility of effective price. The prediction function not only aims at the continuous volatility, but also has the remarkable prediction ability to the jump volatility, but the former is more remarkable.Wu Yanhua and Shi Yufeng[13] studied the effect of true and false jumps on the prediction of volatility by threshold technique. The results show that true jumps have significant effect on the prediction of volatility, but false jumps do not.Cai Guanghui and Ying Xuehai[14] studied the prediction of high-frequency volatility from the perspective of jump, good and bad volatility.They used a more robust ADS detection method to identify jumps, and introduced a Markov state transition mechanism into the Har Model.The empirical results show that, under the Markov State Transition Mechanism, the jump volatility will restrain the future volatility when the market rises, and the positive and negative impact of the good and bad volatility will be balanced when the market rises, in a falling market, good volatility suppresses future volatility, while bad volatility exacerbates future volatility.Gong Yizhou and Huang Ran[15] built a high-frequency volatility model based on the sign jump variation of external information shocks, and they found that the model could predict the high-frequency volatility more accurately according to the type of external information shocks. The above research shows that the jump is an important factor affecting the volatility.

At present, there is not much research on the volatility of Shanghai securities index, only some forecasting research based on Arima model and Garch family model. Therefore, it is necessary to study the volatility of Shanghai securities index.The following structure is arranged as follows: the Second Section introduces the theory; the Third Section Constructs the model; the Fourth Section Empirical Analysis; the Fifth Section summarizes the main research conclusions.

2. Theoretical framework

2.1 Realized volatility model and its decomposition

It is assumed that the logarithmic price P of financial assets follows the semimmartingale process:

$$dP_t = u(t)\,dt + \sigma(t)\,dW(t) + k(t)\,dq(t) \tag{1}$$

Where u (t) is the continuous and locally bounded drift term, σ(t) is the random volatility process, W (t) is the standard Brownian motion, k(t) refers to the size of the corresponding jumps., q (t) is the counting process.

The logarithmic rate of return of financial assets within the time interval δ can be expressed as:

$$r(x+\Delta) = P(x+\Delta) - P(x) \tag{2}$$

According to the quadratic power variation theorem, the quadratic variation process of logarithmic return rate can be expressed as:

$$QV_t = \int_0^t \sigma^2(s)ds + \sum_{s=1}^{N} k^2(s) \tag{3}$$

Where the first term on the right of the equation represents a continuous volatility, the second term represents a jump volatility, n is the number of jumps in the price of a financial asset, and k (s) is the jump range.

According to Andersen et al[1], the realized volatility of a financial asset on day T can be expressed as:

$$RV_t = \sum_{i=1}^{M} r_{t,i}^2 \tag{4}$$

Where $r_{t,i}$ represents the logarithmic return on financial assets at the interval i of Day t, and m represents the total sample of financial assets at day t. According to the study of Barndorff-Nielsen et al[16], when the sampling frequency 1/M is infinitesimal and there is no jump, the realized volatility is a consistent estimate of the integral volatility.

Barndorff-nielsen et al[6] further proposed the quadratic variation theory:

$$BV_t = u_1^{-2} \sum_{j=2}^{M} |r_{t,j}||r_{t,j-1}| \tag{5}$$

Where, when asset prices obey Equation (1), the quadratic variation converges to the integral volatility according to probability. At this point, jump volatility can be expressed as:

$$\sum k^2(s) = RV_t - BV_t \tag{6}$$

According to the statistics constructed by Andersen et al[9,11] :

$$Z_t = \frac{CJ_t / RV_t}{\sqrt{((\pi/2)^2 + \pi - 5)\frac{1}{M}\max\left(1, \frac{RTV_t}{BV_t^2}\right)}} \tag{7}$$

Where,

$$CJ_t = RV_t - BV_t, RTV_t = M u_{4/3}^{-3} \frac{M}{M-2} \sum_{j=3}^{M} |r_{t,j}|^{4/3} |r_{t,j-1}|^{4/3} |r_{t,j-2}|^{4/3} \tag{8}$$

$$u_k = 2^{k/2}\Gamma((k+1)/2)/\Gamma(1/2) \tag{9}$$

$\Gamma(\bullet)$ is the gamma function.

For a given confidence level, the jump is considered to be significant when the Z value is greater than the critical value under the standard normal distribution. So the jump part and the continuous part can be expressed as:

$$CJ_t = I(Z_t > \Phi_\alpha)(RV_t - BV_t) \tag{10}$$

$$C_t = I(Z_t < \Phi_\alpha) RV_t + I(Z_t > \Phi_\alpha) BV_t \tag{11}$$

$I(\bullet)$ is a sign function

2.2 revision of realized volatility

The high-frequency rate of return of financial assets is affected by the market microstructure noise, so that the (4)-type estimator is not a consistent estimate of the real volatility. When the sampling frequency is high enough, the realized volatility will approach the integral volatility gradually, but the noise will also increase with the increase of the sampling frequency, improve the validity of the estimate.Bandi and Russell[4] found that a five-minute sampling rate worked better. Another method of noise reduction is to correct the estimation error caused by noise interference. In this paper, the kernel estimator is used to modify the RV:

$$RV_t^q = \sum r_{t,i}^2 + 2\sum_{h=1}^{q}\left(1-\frac{h}{q+1}\right)\sum_{j=1}^{M-h} r_{t,j} r_{t,j+h} \tag{12}$$

Where q is a nonnegative integer not greater than 4（M/100）2/9 and h is a nonnegative integer not greater than q.In this paper, we choose q = 1, then the expression can be transformed into:

$$RV_t^q = \sum r_{t,i}^2 + \sum_{j=1}^{M-1} r_{t,j} r_{t,j+1} \tag{13}$$

3. Modeling

3.1 HAR-RV model

Corsi constructed the HAR-RV model based on the Heterogeneous Market Hypothesis:

$$RV_t = \alpha + \beta_d RV_{t-1} + \beta_w RVW_t + \beta_m RVM_t + \varepsilon_t \tag{14}$$

Where εt is a random disturbance subject to a positive distribution, RV_{t-1}、RVW_t、 RVM_t is the mean of the realized volatility of the previous day, the previous week and the previous month. This model can capture the long memory of volatility and reflect the trading behavior of different traders in heterogeneous markets.

3.2 HAR-RV-CJ model

Anderson et al [9, 11] constructed the Z statistic to identify the jump and continuous volatility, and then constructed the HAR-RV-CJ model:

$$RV_t = \alpha + \beta_d RV_{t-1} + \beta_w RVW_t + \beta_m RVM_t + \beta_j CJ_t + \varepsilon_t \tag{15}$$

3.3 HAR-S-RV-CJ-CSJV model

Barndorff-Nielsen et al[7] decompose the returns into realized positive (RS^+) and negative (RS^-) semivariances based on the positive and negative intraday returns:

$$RS^+ = \sum_{i=1}^{M} r_{t,i}^2 I(r_{t,i} > 0) \tag{16}$$

$$RS^- = \sum_{i=1}^{M} r_{t,i}^2 I(r_{t,i} < 0) \tag{17}$$

Patton and Sheppard [17] based on this construct the symbolic jump variation:

$$SJV^{+} = (RS^{+} - RS^{-}) I(RS^{+} > RS^{-}) \tag{18}$$

$$SJV^{-} = (RS^{+} - RS^{-}) I(RS^{+} < RS^{-}) \tag{19}$$

On this basis, the extended HAR-RV model including the sign jump variation is constructed:

$$RV_{t} = \alpha + \beta_{d} RV_{t-1} + \beta_{w} RVW_{t} + \beta_{m} RVM_{t} + \beta_{cj} CJ_{t} + \beta_{sjv} SJV_{t}^{+} + \varepsilon_{t} \tag{20}$$

4. Empirical Analysis

4.1 Selection of data sample

In this paper, we use high frequency trading data within 5 minutes of the Shanghai securities 50 index from May 25,2020 to September 30,2020. There are 15,984 sample data, the sample data is from the tonghuashun. Take the logarithm of the closing price of shanghai securities 50 index to find the logarithmic return rate. In order to improve the accuracy of empirical results, the logarithmic return rate was expanded by 100 times, namely: $r_{t,i} = 100(\ln P_{t,i} - \ln P_{t,i-1})$

Then the corresponding sequence diagram is drawn according to the sample. As can be seen from figure 1(a) , the Shanghai securities 50 index rose in the middle of 2020 and only started to fall back in 2021.This was due to our country's good precautionary measures, which brought the epidemic under control in just a few months and led to a rapid economic recovery, the stock market, as a barometer of the economy, has also been allowed to rise. Coupled with loose monetary policy, a large number of capital inflows into the stock market, the index naturally rose.

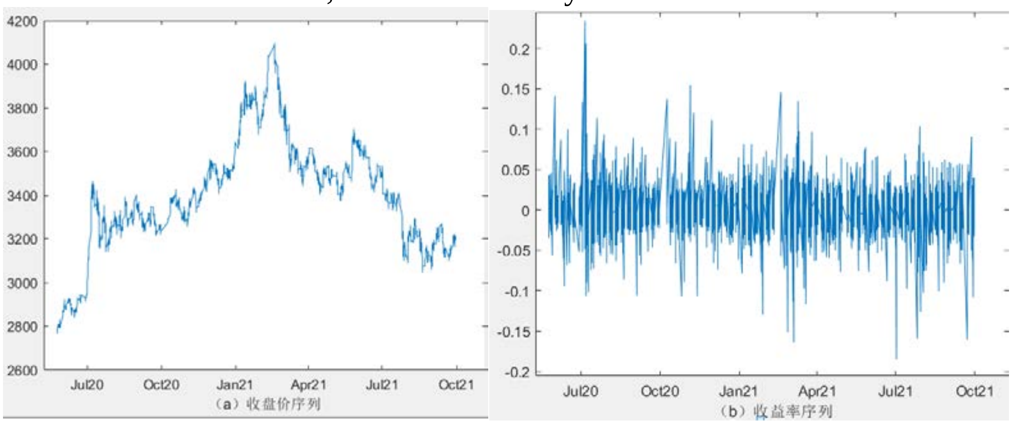

Figure 1. closing price and yield series of the Shanghai 50 index

4.2 statistical description and analysis of data

Table 1. descriptive statistics of explanatory variables

statistic	RV	RVD	RVW	RVM	CJ	SJV+
Mean	1.349539	1.349308	1.340951	1.322854	0.235844	0.0239961
Min	0.143638	0.143638	0.419377	0.721796	0	0
Max	8.918997	8.918997	5.561108	3.150332	3.389734	7.340976
Std	1.110881	1.111054	0.802502	0.55178	0.437071	0.566691
Kurt	17.10425	17.09651	10.503986	4.296206	20.27649	85.82131
Skew	3.056003	3.054856	2.392674	1.411761	3.656615	7.530809
ADF	-7.21***	-7.17***	-3.48**	-3.43**	-13.87***	-16.80458***

Note: ** means significant at 5% level, *** means significant at 1% level

It can be seen from Table 1 that the t-value of the unit root test of each variable is significant at a level above 5%, indicating that these sequences are stable.indicating that these data have typical characteristics of financial data with sharp peaks, thick tails and right skewness.

4.3 Analysis of empirical results

Table 2. Har-RV-CJ model regression results

coefficience	estimator	st.d	t value	p value
α	0.19546 *	0.099709	1.9603	0.0508
β_d	0.20589***	0.046468	4.4307	0
β_w	0.2485 ***	0.072747	3.4159	0
β_m	0.112529	0.085075	1.4727	0.1418
β_{cj}	1.6026***	0.089764	17.853	0

As can be seen from Table 2, the coefficients of daily effect, weekly effect and monthly effect are 0.20589, 0.2485 and 0.112529 respectively, among which the daily effect and weekly effect are significant at 99% confidence, indicating that realized volatility has strong autocorrelation.Among the three, the estimated value and significance of the weekly effect are the highest, indicating that the realized volatility has the strongest memory for the weekly average volatility.If the daily effect, weekly effect and monthly effect are regarded as the market's response to the investment behaviors of short-term investors, medium-term investors and long-term investors, it can be seen that medium-term investors account for a relatively large proportion in the market, followed by short-term investors and long-term investors. Based on the efficient market hypothesis, markets quickly reflect information into prices, meaning that an efficient market should have a very short memory. Therefore, this shows that the development of China's financial market is not mature and needs to be improved.

Table 3. Estimation results of extended HAR-RV-CJ model

coefficience	estimator	std	t value	p value
α	0.17559 *	0.097541	1.8002	0.07
β_d	0.17246***	0.046179	3.7345	0
β_w	0.27039 ***	0.071286	3.7931	0
β_m	0.13436	0.083146	1.6159	0.107
β_{cj}	1.3751***	0.10491	13.107	0
β_{sjv}	0.32169***	0.08142	3.951	0

Note: * represents significant at 10% level, ** represents significant at 5% level, and *** represents significant at 1% level

As can be seen from Table 3, the daily effect is decreasing, while the weekly effect is increasing, while the monthly effect is basically unchanged. Among all explanatory variables, the coefficient and significance of jump variable are always the highest, which indicates that jump has a strong explanatory ability to volatility and is an important explanatory variable of volatility prediction. The goodness of fit of the model changes from 0.652 to 0.669 with the introduction of symbolic jump, which indicates that symbolic jump can optimize HAR-RV-CJ model and improve the explanatory ability of the model. In addition, the coefficient of sign jump is 0.32, which is larger than the coefficient of daily, weekly and monthly effects, indicating that the variation of sign jump itself is also an important explanatory variable.

5. Conclusion

This paper estimates the volatility of the Shanghai securities 50 index using five minute trading data from May 25,2020, to 2021 September 30,2020, and revises the volatility using nuclear estimator. Then, the volatility is decomposed into continuous part and jump part by quadratic power variation. Finally, HAR-RV-CJ model and its extended model are used to study the effect of jump on the volatility of Shanghai securities 50 Index.

Realized volatility, jump volatility and symbol jump sequence all have the characteristic of peak fat tail and right deviation. Moreover, the percentage of daily jump in Shanghai securities 50 index is high, which has a strong positive explanatory effect on realized volatility.The introduction of symbolic jump strengthens the interpretation ability of the HAR-RV-CJ model, which makes the weekly effect stronger, the solar system effect weaker and the lunar effect unchanged.

Based on the HAR-RV-CJ model, we find that the week effect is stronger than the day effect and the month effect, which shows that the proportion of medium-term investors is higher, followed by short-term investors and long-term investors. China's financial market is still immature and needs to be improved.

In the volatile financial market, volatility estimation is particularly important. Extracting various components of volatility from high-frequency data can help investors improve the accuracy of volatility prediction and reduce investment risks. At the same time, the study of volatility can help market management departments make better plans to improve the market.

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